

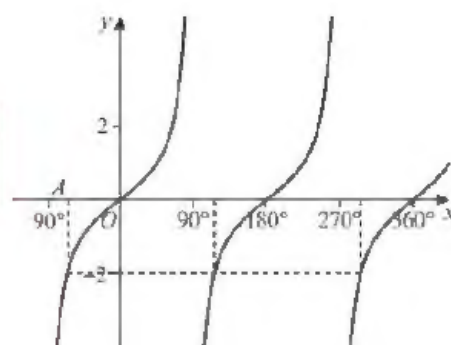
**Exercise 10D**

1 The diagram shows a sketch of $y = \tan x$.

- a Use your calculator to find the principal solution to the equation $\tan x = -2$.
- b Use the graph and your answer to part a to find solutions to the equation $\tan x = -2$ in the range $0 \leq x \leq 360^\circ$.

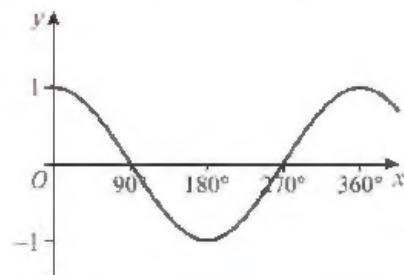
Hint

The principal solution is marked A on the diagram.



2 The diagram shows a sketch of $y = \cos x$.

- a Use your calculator to find the principal solution to the equation $\cos x = 0.4$.
- b Use the graph and your answer to part a to find solutions to the equation $\cos x = \pm 0.4$ in the range $0 \leq x \leq 360^\circ$.



3 Solve the following equations for θ , in the interval $0 < \theta \leq 360^\circ$:

- a $\sin \theta = -1$ b $\tan \theta = \sqrt{3}$ c $\cos \theta = \frac{1}{2}$
 d $\sin \theta = \sin 15^\circ$ e $\cos \theta = -\cos 40^\circ$ f $\tan \theta = -1$
 g $\cos \theta = 0$ h $\sin \theta = -0.766$

Hint

Give your answers exactly where possible, or round to 3 significant figures.

4 Solve the following equations for θ , in the interval $0 < \theta \leq 360^\circ$:

- a $7 \sin \theta = 5$ b $2 \cos \theta = -\sqrt{2}$ c $3 \cos \theta = -2$ d $4 \sin \theta = -3$
 e $7 \tan \theta = 1$ f $8 \tan \theta = 15$ g $3 \tan \theta = -11$ h $3 \cos \theta = \sqrt{5}$

5 Solve the following equations for θ , in the interval $0 < \theta \leq 360^\circ$:

- a $\sqrt{3} \sin \theta = \cos \theta$ b $\sin \theta + \cos \theta = 0$ c $3 \sin \theta = 4 \cos \theta$
 d $2 \sin \theta - 3 \cos \theta = 0$ e $\sqrt{2} \sin \theta = 2 \cos \theta$ f $\sqrt{5} \sin \theta + \sqrt{2} \cos \theta = 0$

6 Solve the following equations for x , giving your answers to 3 significant figures where appropriate, in the intervals indicated:

- a $\sin x = -\frac{\sqrt{3}}{2}$, $-180^\circ \leq x \leq 540^\circ$ b $2 \sin x = -0.3$, $-180^\circ \leq x \leq 180^\circ$
 c $\cos x = -0.809$, $-180^\circ \leq x \leq 180^\circ$ d $\cos x = 0.84$, $-360^\circ < x < 0^\circ$
 e $\tan x = -\frac{\sqrt{3}}{3}$, $0 \leq x \leq 720^\circ$ f $\tan x = 2.90$, $80^\circ \leq x \leq 440^\circ$

E/P

- 7 A teacher asks two students to solve the equation $2 \cos x = 3 \sin x$ for $-180^\circ \leq x \leq 180^\circ$. The attempts are shown:

Student A:

$$\tan x = \frac{3}{2}$$

$$x = 56.3^\circ \text{ or } x = -123.7^\circ$$

Student B:

$$4 \cos^2 x = 9 \sin^2 x$$

$$4(1 - \sin^2 x) = 9 \sin^2 x$$

$$4 = 13 \sin^2 x$$

$$\sin x = \pm \sqrt{\frac{4}{13}}, x = \pm 33.7^\circ \text{ or } x = \pm 146.3^\circ$$

- a Identify the mistake made by Student A. (1 mark)
- b Identify the mistake made by Student B and explain the effect it has on their solution. (2 marks)
- c Write down the correct answers to the question. (1 mark)

- 8 a Sketch the graphs of $y = 2 \sin x$ and $y = \cos x$ on the same set of axes ($0 \leq x \leq 360^\circ$).
b Write down how many solutions there are in the given range for the equation $2 \sin x = \cos x$.
c Solve the equation $2 \sin x = \cos x$ algebraically, giving your answers to 1 d.p.

E/P

- 9 Find all the values of θ , to 1 decimal place, in the interval $0 < \theta < 360^\circ$ for which $\tan^2 \theta = 9$. **(5 marks)**

Problem-solving

When you take square roots of both sides of an equation you need to consider both the positive and the negative square roots.

E/P

- 10 a Show that $4 \sin^2 x - 3 \cos^2 x = 2$ can be written as $7 \sin^2 x = 5$. **(2 marks)**
b Hence solve, for $0 \leq x \leq 360^\circ$, the equation $4 \sin^2 x - 3 \cos^2 x = 2$.
Give your answers to 1 decimal place. **(7 marks)**

E/P

- 11 a Show that the equation $2 \sin^2 x + 5 \cos^2 x = 1$ can be written as $3 \sin^2 x = 4$. **(2 marks)**
b Use your result in part a to explain why the equation $2 \sin^2 x + 5 \cos^2 x = 1$ has no solutions. **(1 marks)**